Testing Outerplanarity With One-Sided Error

and other forbidden minors

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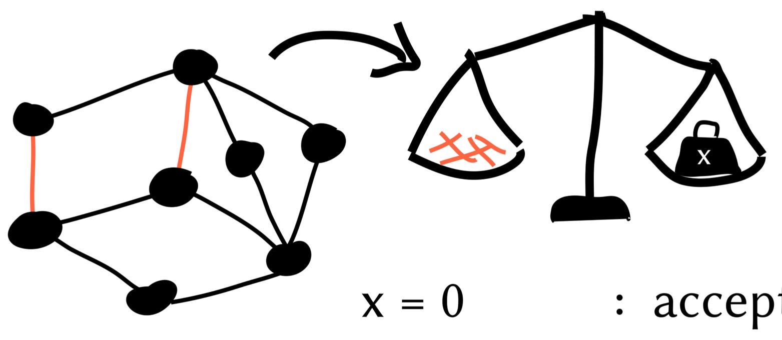
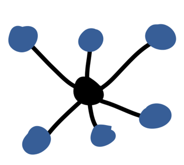
Property Testing: Outerplanarity



x = 0 : accept always

0 < x ≤ εdn : don't care

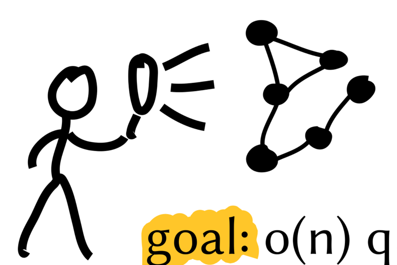
εdn < x : reject w.p. 2/3



#● < d

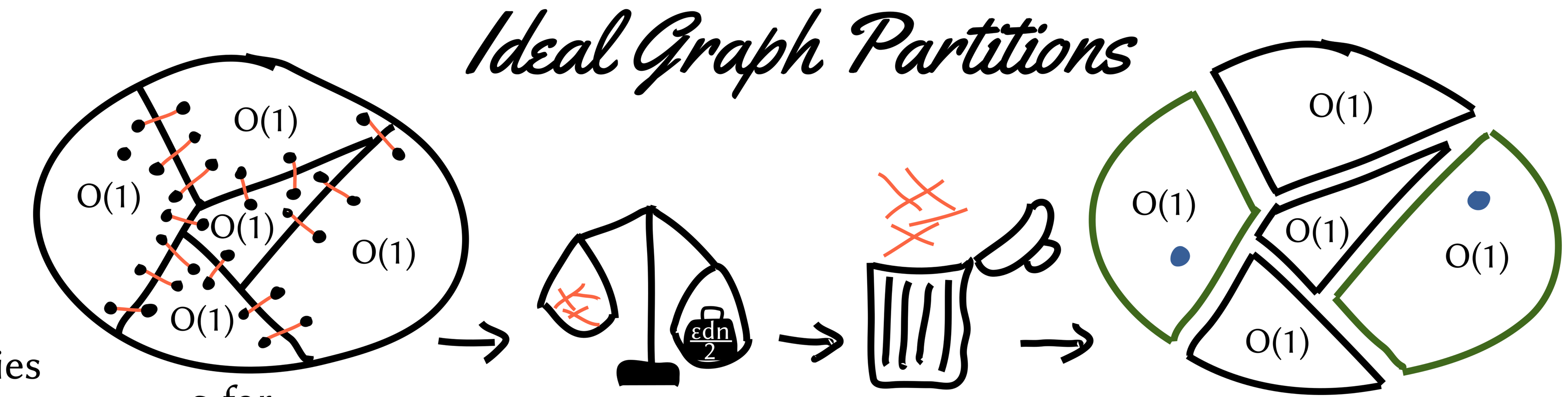
ε-far

ε-close



goal: o(n) queries to adjacency lists

Ideal Graph Partitions



ideal tester: 1. draw O(ε) vertices

2. reconstruct parts locally

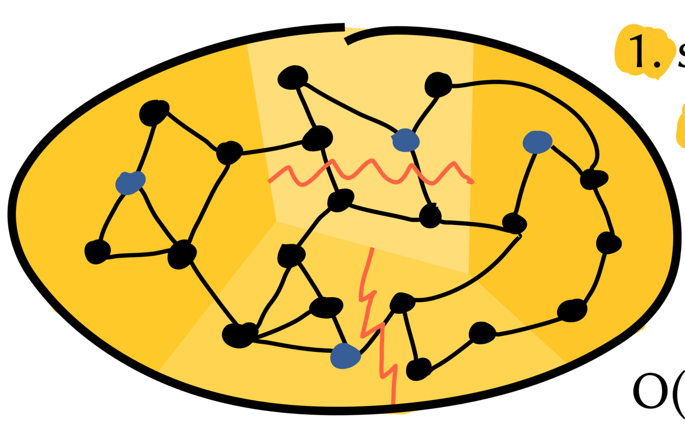
3. check parts for property

but reality isn't ideal...

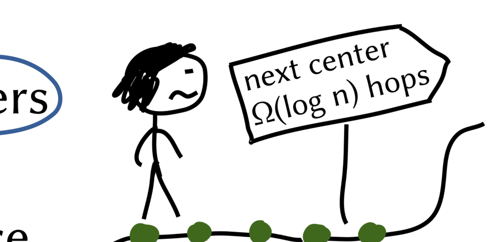
Main Theorem

Testing $\mathscr{F}$-minor-freeness for any $\mathscr{F}$ that contains a $K_{2,k}$ (or k×2-grid or k-circus) has query complexity $O(n^{2/3} / \varepsilon^5)$.

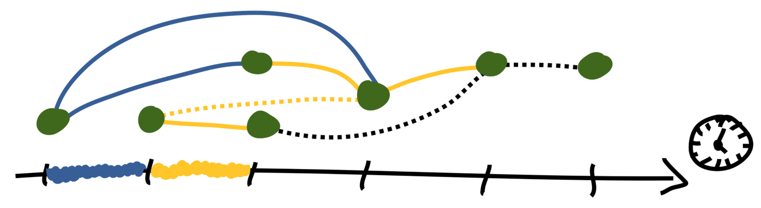
A Global Voronoi Partition...



1. select $\Theta(n^{2/3})$ random centers
2. construct Voronoi cells according to path distance
3. sort out remote vertices
4. shatter Voronoi cells into $O(n^{2/3})$ *core* clusters of size $O(n^{1/3})$



Remote Clusters



1. each remote vertex picks random delay
2. after delay, start BFS: one level per time
3. construct *remote* clusters from BFS

Checklist

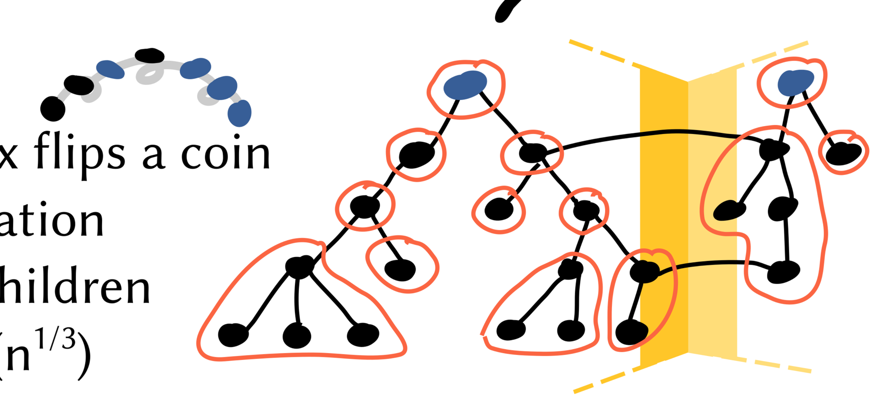
☑ locally reconstructable

☑ cut ●)-(● ∈ O(εn)

☑ cut ●)-(● ∈ O(εn)

☐ cut ●)-(● ∉ O(εn)

...Constructed Locally



1. each vertex flips a coin
2. BFS exploration
3. cut heavy children

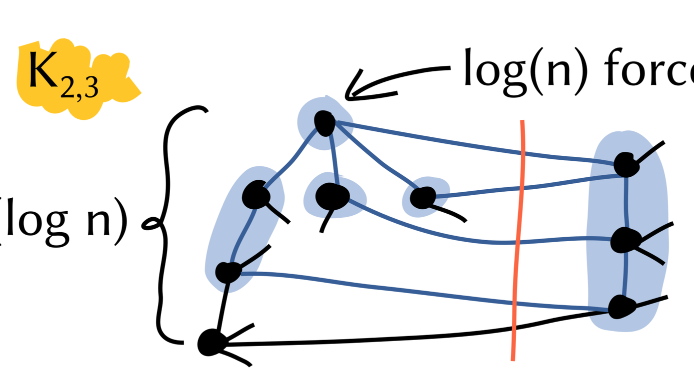
complexity: $O(n^{1/3})$

What if the cut between core clusters is large?

Then there exists a $K_{2,k}$-minor in this cut!

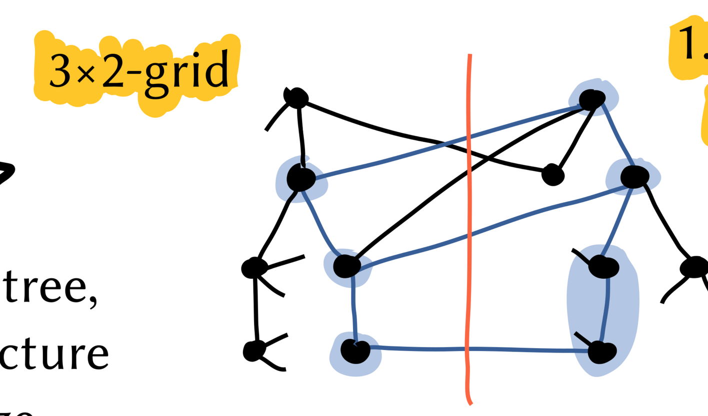
Cut Separability

Theorem: cuts of size > f between core clusters imply {$K_{2,k}$, k×2-grid, k-circus}-minors



$f \approx \Theta(k\ d \log(n))$

idea: always have BFS tree, enforce more structure by large cut size



$f \approx \Theta(d^{d^{k^2}})$

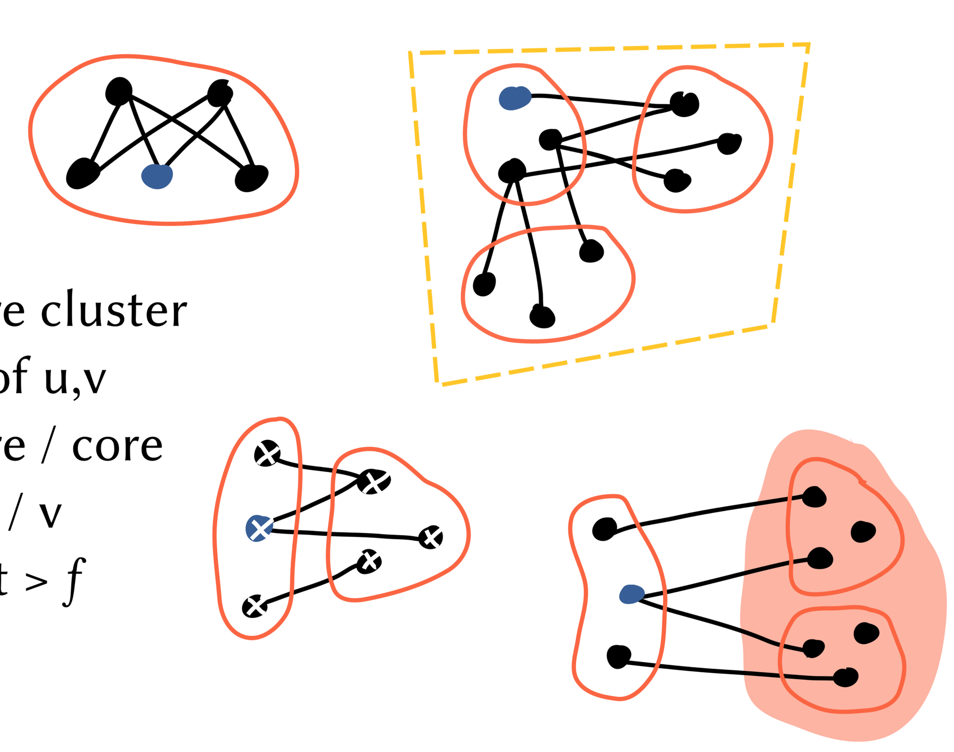
1. f enforces d^{k^2}-path on left side
2. f enforces k^2-path on right side
3. Hall matching & Erdős–Szekeres theorem imply k non-intersecting cut edges between paths

So, if all cuts between core clusters are smaller than f, we can remove them?

Well, there are too many of them... but basically, yes.

The Algorithm

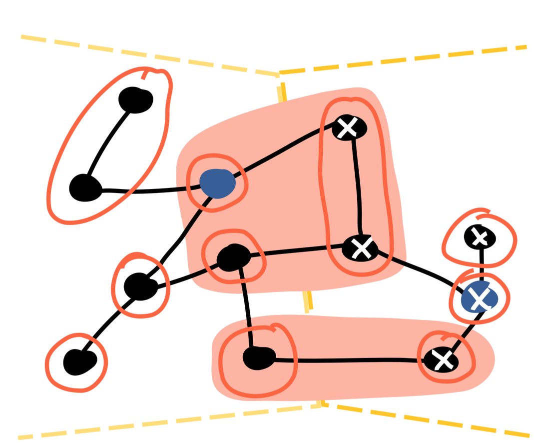
1. sample $O(f / \varepsilon)$ edges
2. for every sampled edge (u,v):
   - i) explore cluster(s) of u,v
   - ii) compute cut sizes between core cluster and remaining Voronoi cell of u,v
   - iii) compute cut sizes between core / core and core / super cluster of u / v
3. reject iff minor found or some cut > f



Super Core Clusters

Problem: $f \cdot \#(\text{core clusters})^2 \notin O(\varepsilon dn)$

1. mark each Voronoi cell w.p. $1/n^{1/3}$
2. mark each core cluster of marked cells
3. join unmarked core clusters with marked neighboring core clusters



~~☑ locally reconstructable~~

☑ local membership queries

☑ $f \cdot \#(\text{core clusters}) \cdot \#(\text{super clusters}) \in O(\varepsilon dn)$

This work will appear on ICALP 2018.

Partitions based on: Lenzen and Levi, ICALP 2018

Recent progess: Kumar, Seshadhri and Solman, ECCC 2018

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