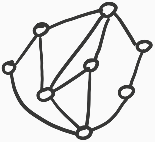
Testable Properties in General Graphs and Random Order Streaming

Artur Czumaj, *Hendrik Fichtenberger*, Pan Peng, Christian Sohler

Property Testing: in a Nutshell



planar ✓

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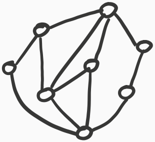

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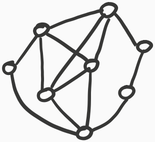

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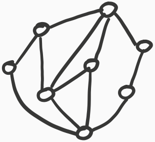

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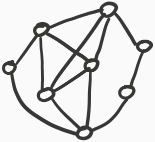

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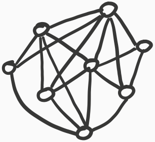

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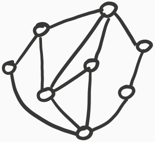

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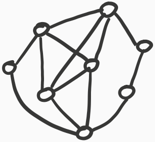

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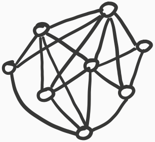

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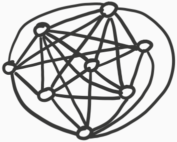

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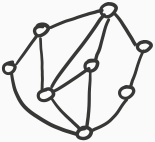

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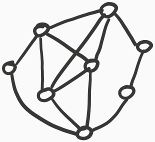

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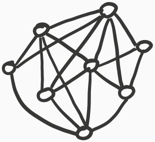

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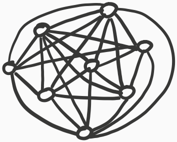

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time complexity: $\Omega(|V|)$

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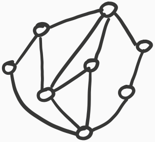

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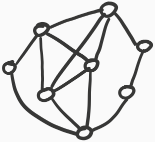

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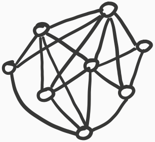

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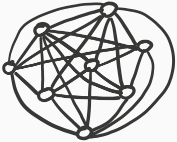

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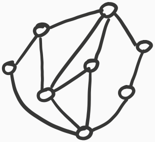

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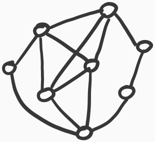
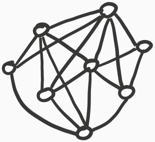
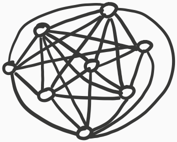


slightly non-planar ✗



quite non-planar ✗



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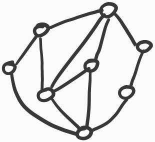
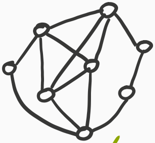
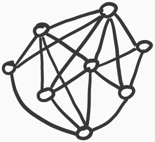


planar ✓



slightly non-planar ✓/✗



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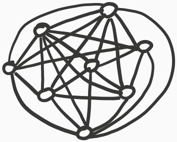

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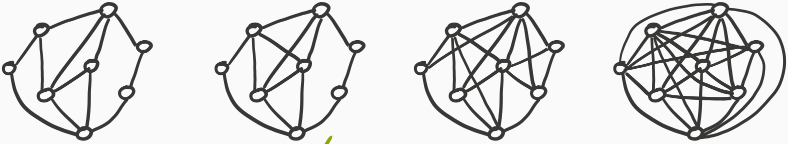
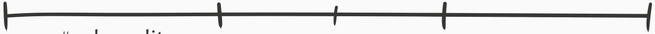


planar ✓ non-planar ~~✓~~ ✗ non-planar ✗ non-planar ✗



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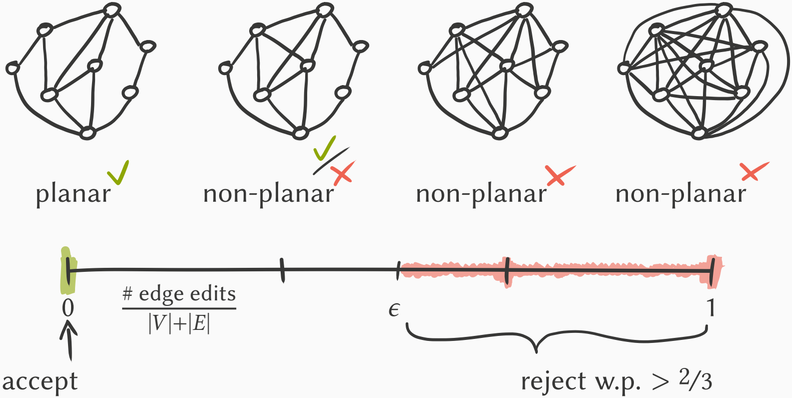

planar ✓

non-planar ~~✓~~ ✗

non-planar ✗

non-planar ✗

0

$\frac{\text{\# edge edits}}{|V|+|E|}$

ϵ

1

accept

reject w.p. > ²⁄₃

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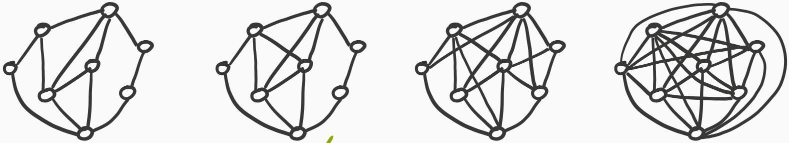
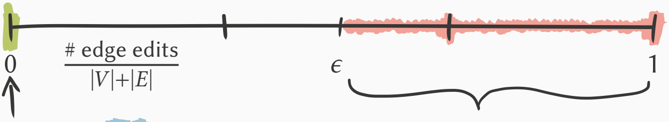


planar ✓ | non-planar ~~✓~~ ✗ | non-planar ✗ | non-planar ✗



0

$\frac{\text{\# edge edits}}{|V|+|E|}$

ϵ

1

accept w.p. $>$ $2/3$

reject w.p. $>$ $2/3$

1-sided error / 2-sided error

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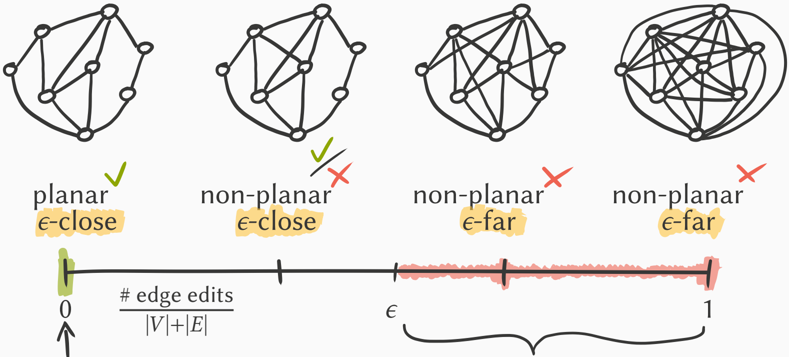

planar ✓
ϵ-close

non-planar ~~✓~~ ✗
ϵ-close

non-planar ✗
ϵ-far

non-planar ✗
ϵ-far

$\frac{\#\text{ edge edits}}{|V|+|E|}$

0

ϵ

1

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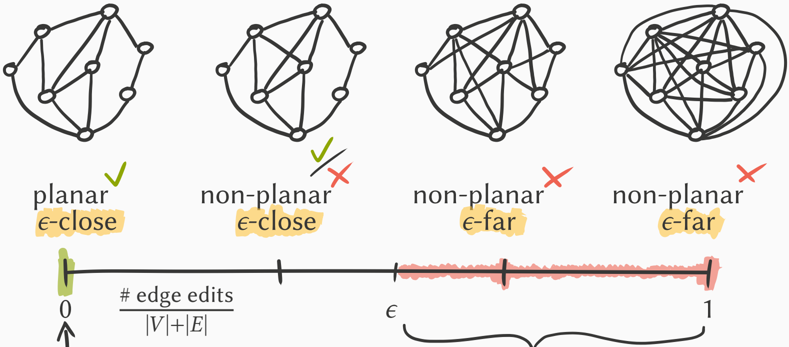

planar ✓

ϵ-close

non-planar ~~✓~~ ✗

ϵ-close

non-planar ✗

ϵ-far

non-planar ✗

ϵ-far

0 $\qquad \frac{\#\text{ edge edits}}{|V|+|E|} \qquad \epsilon \qquad 1$

accept w.p. $> {}^2\!/_3$

reject w.p. $> {}^2\!/_3$

1-sided error / 2-sided error

complexity: # queries to data structure

The Bounded-Degree Model

☒ bounded-degree model: $\forall v \in V : d(v) \leq d, d \in O(1), n := |V|$

☒ input structure: adjacency lists (1 query $\hat{=}$ 1 entry)

☒ error: 1-sided

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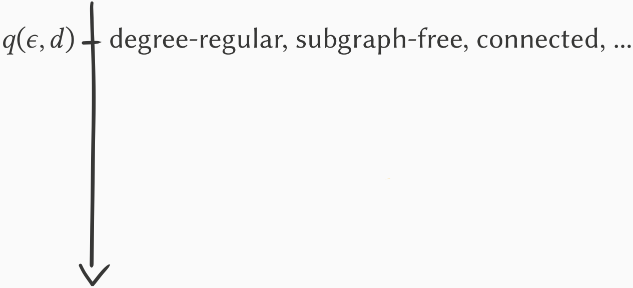

$q(\epsilon, d)$ degree-regular, subgraph-free, connected, ...

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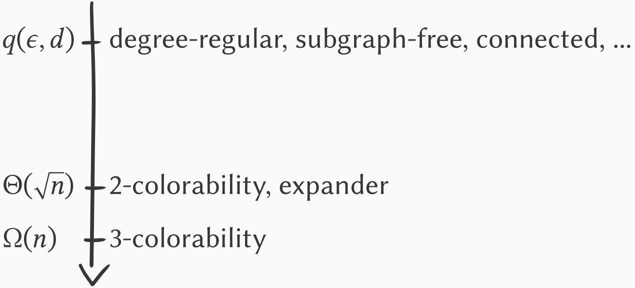

$q(\epsilon, d)$ — degree-regular, subgraph-free, connected, ...

$\Theta(\sqrt{n})$ — 2-colorability, expander

$\Omega(n)$ — 3-colorability

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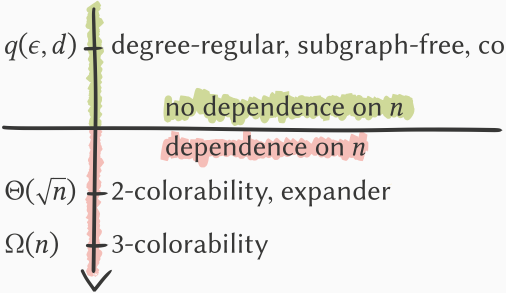


$q(\epsilon, d)$ degree-regular, subgraph-free, connected, ...

no dependence on n

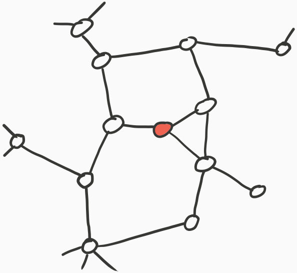
dependence on n

$\Theta(\sqrt{n})$ 2-colorability, expander

$\Omega(n)$ 3-colorability

q-Disks / q-Hop Neighborhoods

$\mathrm{disk}_q(v)$: unlabelled subgraph induced by $\mathrm{BFS}(v)$ of depth q



q-Disks / q-Hop Neighborhoods

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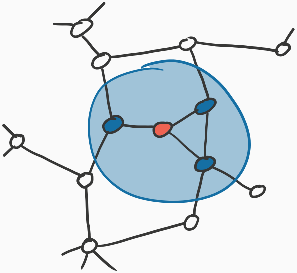

disk_1(●)

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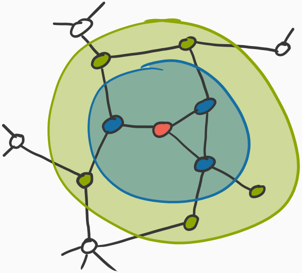
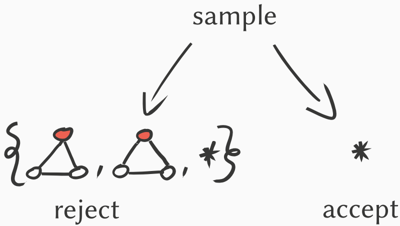


$\text{disk}_1($●$)$

$\text{disk}_2($●$)$

Constant-Query Testers



sample

reject

accept

Theorem [GR’09, ...]

A property tester for bounded-degree graphs with constant query complexity $q := q(\epsilon)$ can be transformed into an algorithm that

1. obtains a uniform sample S of $\Theta(q)$ many q-disks
2. rejects iff S is from a family of forbidden sets of q-disks

Goldreich, Ron, STOC’09

The Random-Neighbor Model

☒ bounded-degree model
☒ input structure: adjacency lists
☒ error: 1-sided

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What can a constant-query
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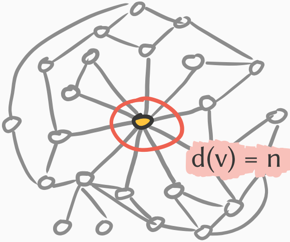

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BFS

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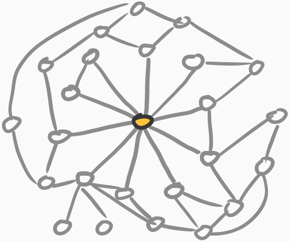

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B~~F~~S

random / subsampling BFS

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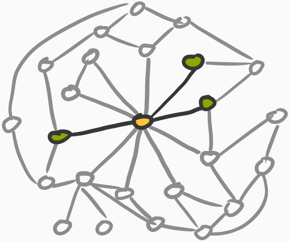

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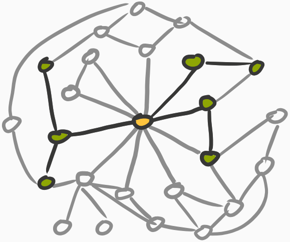

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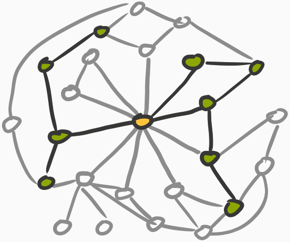

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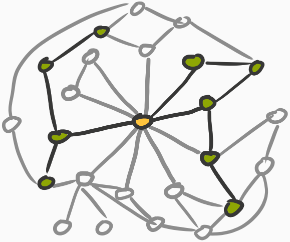

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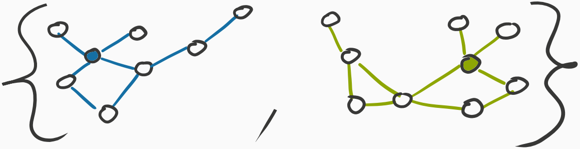
B~~F~~S

random / subsampling BFS

tester obtains at most q many q-disks (with bounded degree q)

Bounded Degree vs. General Graphs



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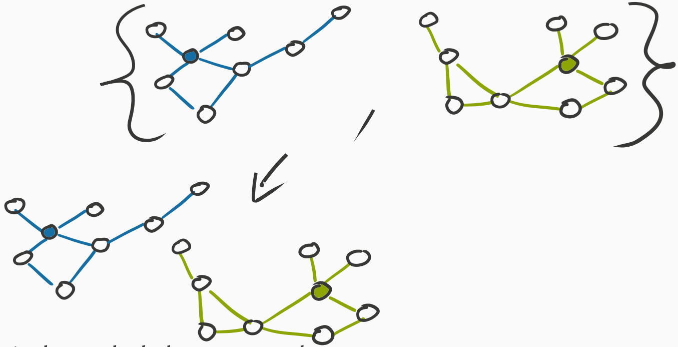

in bounded-degree graphs:
random q-disks disjoint w.h.p.

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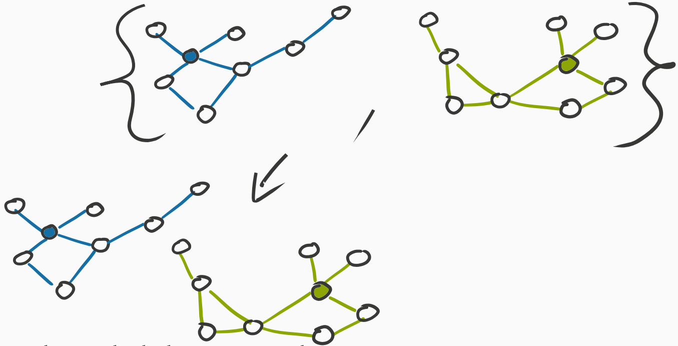

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↓

bijective decomposition ✓
of BFS subgraphs into q-disks

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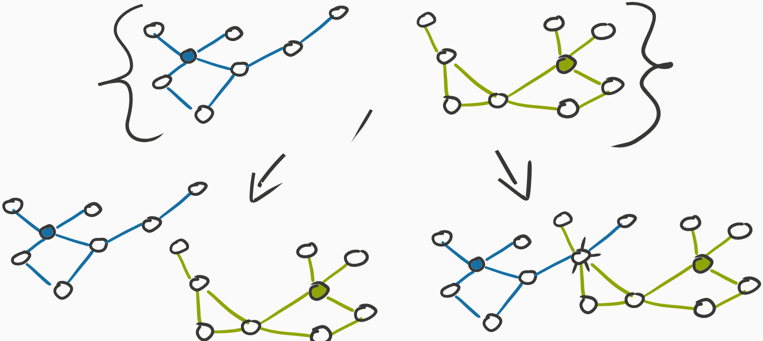

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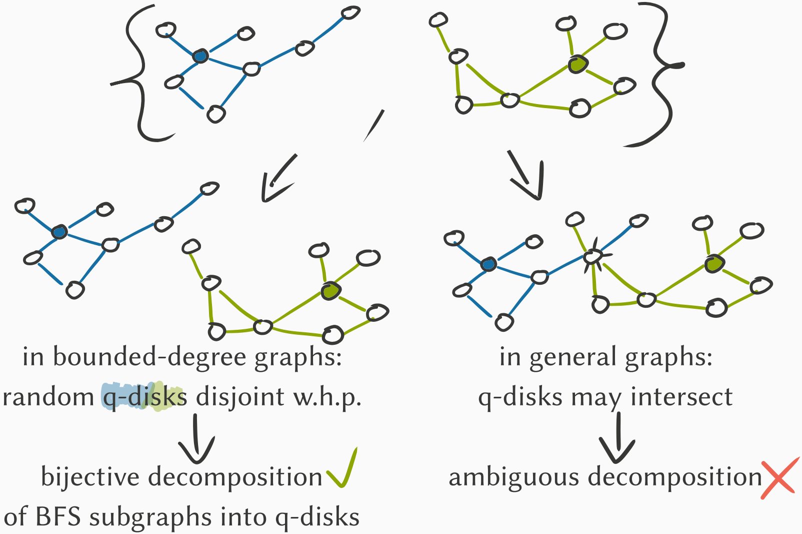

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↓

bijective decomposition ✓
of BFS subgraphs into q-disks

in general graphs:
q-disks may intersect

↓

ambiguous decomposition ✗

The Problem of Intersecting q-Disks

problem: two random BFS visit the same vertex

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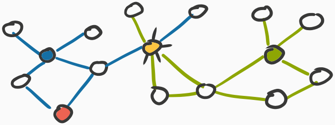

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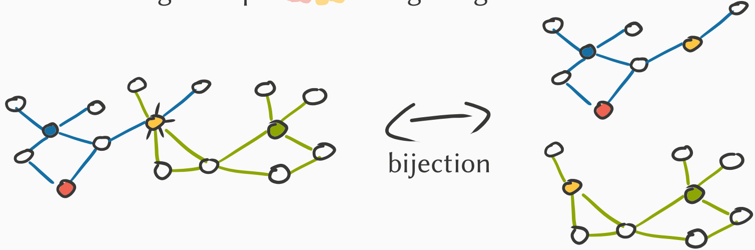

bijection

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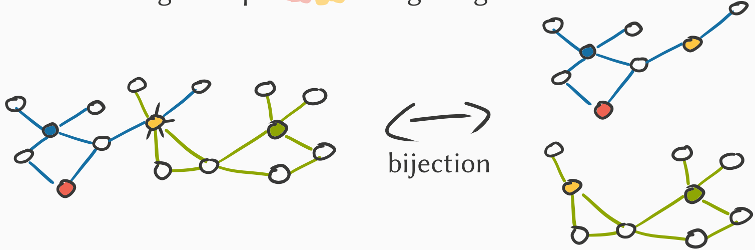
only likely for $\underbrace{\text{linear degree vertices}}_{O(1)\text{ many}}$

solution: assign unique color to high degree vertices



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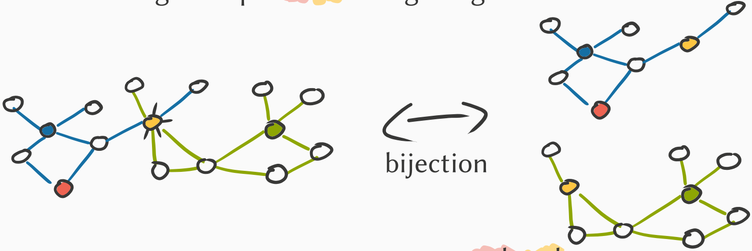

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Constant-Query Testers in the Random-Neighbor Model

Theorem

A property tester *for general graphs in the random-neighbor model* with constant query complexity $q := q(\epsilon)$ can be transformed into an algorithm that

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why q-disks and not simply forbidden *subgraphs*?

The Streaming Model

- ☒ general graphs
- ☒ input structure: adjacency lists
- ☒ error: 1-sided

Monemizadeh, Muthukrishnan, Peng, Sohler, ICALP'17; Peng, Sohler, SODA'18

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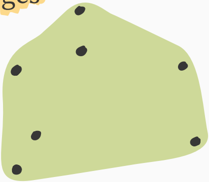

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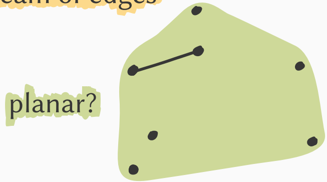

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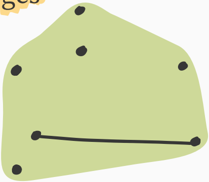

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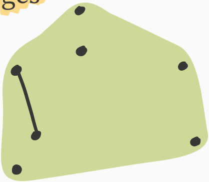

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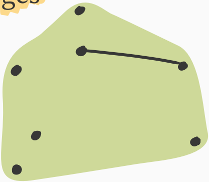

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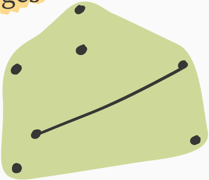

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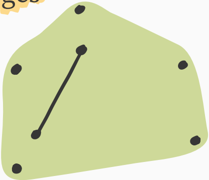

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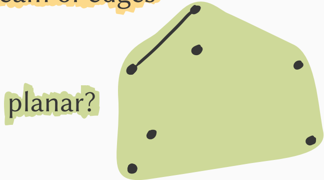

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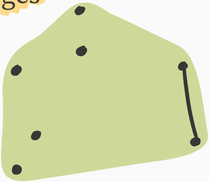

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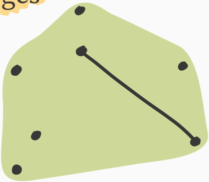

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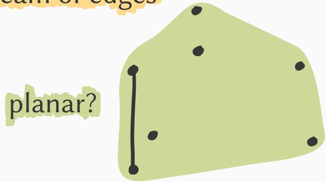

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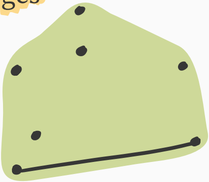

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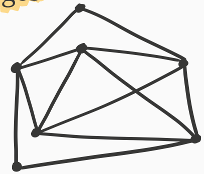

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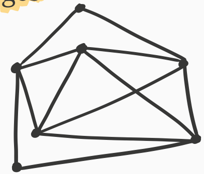

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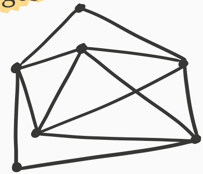

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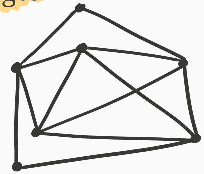

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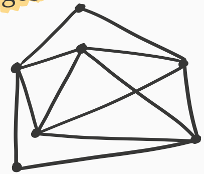

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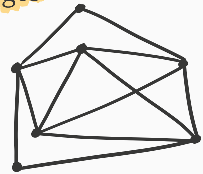

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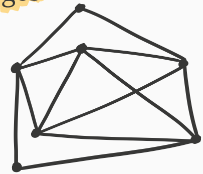

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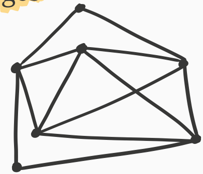

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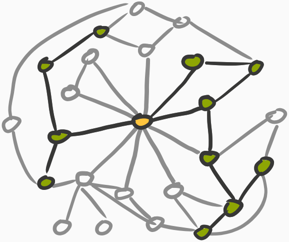

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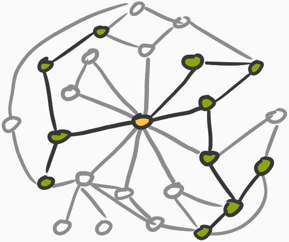

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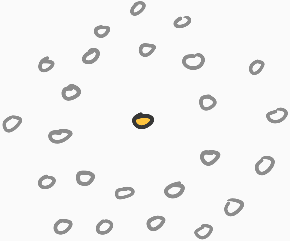

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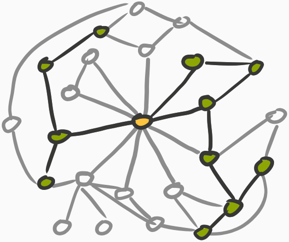

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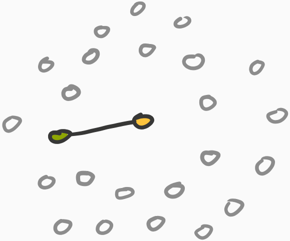

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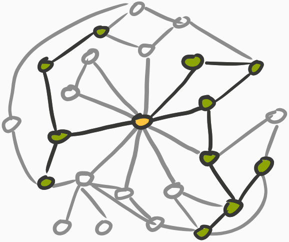

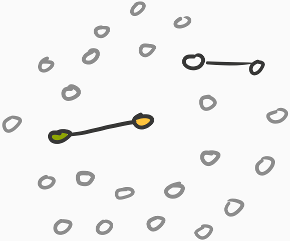

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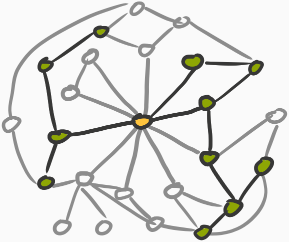

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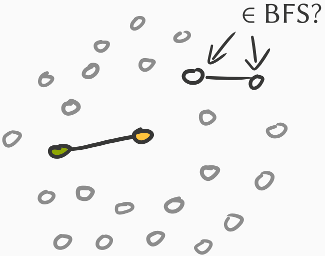

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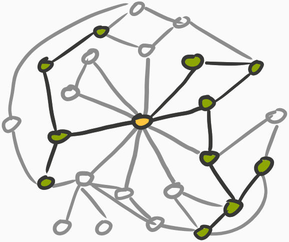

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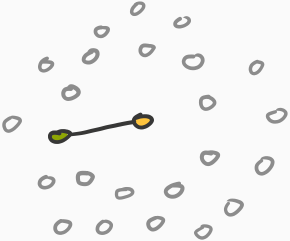

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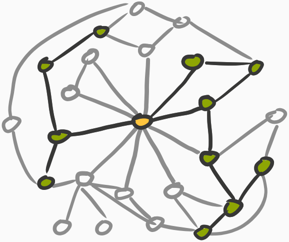

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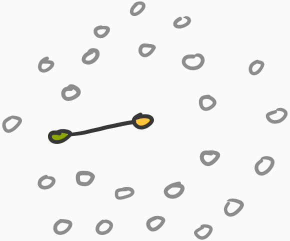

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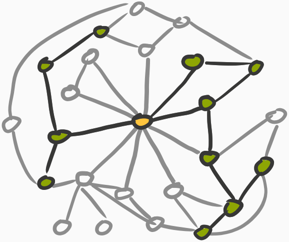

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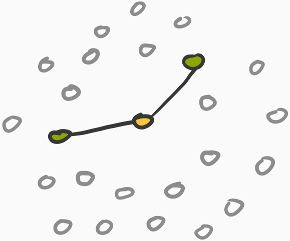

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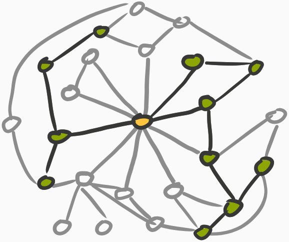

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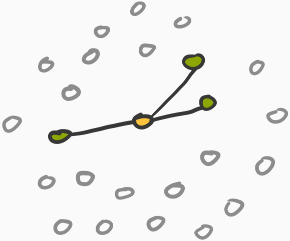

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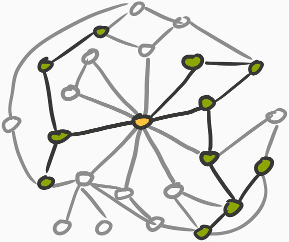

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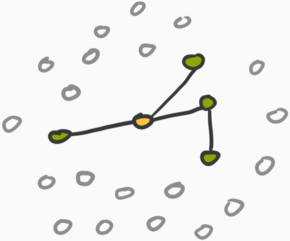

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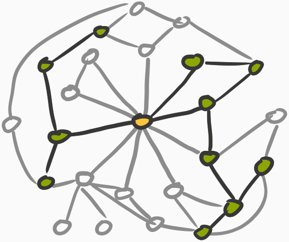

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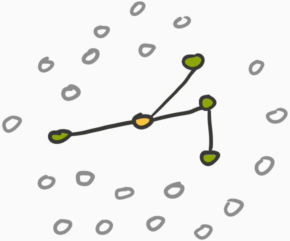

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then: bound probabilities to see colored q-disks independently

From Constant-Query to Streaming Testers

Theorem

Every constant-query property tester for general graphs in the random-neighbor model with one-sided error and constant query complexity admits a $O(\log n)$ space random order streaming tester.

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open problem: similar result for testers with two-sided error